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The Diagnosability of $(K_4 - \{e\})$-free Graphs under the PMC Diagnosis Model

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Abstract. The ability of identifying all the faulty devices in a multiprocessor system is known as diagnosability. The PMC model is the test-based diagnosis with a processor performing the diagnosis by testing the neighboring processors via the links between them. In this paper, we discuss the diagnosability of a $(K_4 - \{e\})$-free graph under the PMC model.

Keywords: system diagnosis, diagnosability, PMC model

1. Introduction

Sensor networks have become increasingly popular in computer technology. There are more than one sensor node or processor in a sensor network. Usually, the sensor nodes, processors and the links are

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modeled as a graph topology. Even slight malfunctions could make the service not work, and thus, the reliability of a system is the key issue. To ensure the reliability of the system, the devices found faulty should be replaced with fault-free ones instantly. Therefore, it is important for the ability to identify any faulty devices, as known as *system diagnosis*. The maximum number of faulty devices which can be identified correctly is *diagnosability*. If all the faulty devices in a system can be pointed out precisely with the faulty devices being t at most, the system is t*-diagnosable*. Many results about the diagnosis and the diagnosability have been proposed [9, 12, 13].

Preparata, Metze, and Chien have proposed the PMC diagnosis model in [10]. The PMC model is the tested-based diagnosis. Under the PMC model, a processor performs the diagnosis by testing the neighboring processors via the links between them. Hakimi and Amin have showed that, a system G is t-diagnosable if G is t-connected with at least $2t+1$ nodes under the PMC model [5]. Furthermore, they gave a necessary and sufficient condition for verifying whether or not a system is t-diagnosable under the PMC model. Some related studies have appeared in the literature [2, 3, 6, 11].

In [8], Lin and Teng proposed the diagnosability for a triangle-free graph under the PMC model. They proved that the diagnosability of a triangle-free graph G is the minimum degree of G if G is not isomorphic to a complete bipartite graph. In this paper, we discuss the diagnosability of a $(K_4 - \{e\})$-free graph under the PMC model. Suppose that G is a $(K_4 - \{e\})$-free graph, and the minimum degree of G is $\delta(G)$. We prove that the diagnosability of G is at most $\delta(G) - 1$ if and only if G contains a subgraph $A + K_1$, where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$ with $deg_G(x) = \delta(G)$ for every node x in A; otherwise, the diagnosability of G is $\delta(G)$.

2. Preliminaries

The layout of processors and links in a multiprocessor system is usually represented by an undirected graph. For graph definitions and notations, we follow [7]. Let $G = (V, E)$ be a graph. The *node set* V is a finite set, and the *edge set* E is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. Two nodes u and v are *adjacent* if $(u, v) \in E$. The *neighbourhood* of a node u in G is the set of all nodes adjacent to u in G, denoted by $N_G(u)$. The *degree* of u in G is denoted by $deg_G(u) = |N_G(u)|$. We use $\delta(G)$ to represent the minimum degree of G. The *induced subgraph* $G[H] = (H, L)$ is the subgraph of G, where the node set H is a subset of V, and the edge set $L = \{(x, y) \mid (x, y) \in E$ for every node $x, y \in H\}$. We use $G + H$ to denote a *join graph* with G and H, where the node set is $V(H) \cup V(G)$, and edge set is $E(G) \cup E(H) \cup \{(x, y) \mid x \in V(G)$ and $y \in V(H)\}$.

Under the PMC model, we assume that the adjacent processors can perform tests on each other. Let $G = (V, E)$ represent the underlying topology of a multiprocessor system. For any two adjacent nodes u and v in G, the ordered pair (u, v) represents a *test* that u diagnoses v. In this situation, u is a *tester*, and v is a *testee*. If u evaluates v to be faulty (respectively, fault-free), the outcome of the test (u, v) is 1 (respectively, 0). The faults considered here are permanent, hence the outcome is *reliable* if and only if the tester is fault-free. A *faulty set* F is the set of all faulty nodes in G. A *test assignment* for a system G is a collection of tests, which can be modeled as a directed graph $T = (V, L)$. The nodes u and v are adjacent in G, and the test $(u, v) \in L$. The collection of all test results from the test assignment T is termed a *syndrome*. A syndrome of T is a mapping $\sigma : L \to \{0, 1\}$. Let $T = (V, L)$ be a test assignment, and F be a subset of V. For any given syndrome σ resulting from T, F is said to be *consistent* with σ if for a test $(u, v) \in L$ such that $u \in V - F$, then $\sigma(u, v) = 1$ if and only if $v \in F$. Thus the fault-free testers always give correct test results, while the faulty testers give rise to unreliable results. Therefore, a faulty set F may be consistent with different syndromes. We use $\sigma(F)$ to represent the set of all possible syndromes with which the faulty set F can be consistent. Let F_1 and F_2 be two distinct faulty sets of V. Then, F_1 and F_2 are *distinguishable* if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$; otherwise, F_1 and F_2 are *indistinguishable*. In other words, (F_1, F_2) is a *distinguishable pair* if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$; otherwise, (F_1, F_2) is an *indistinguishable pair*. Suppose that $|F_1| \leq t$ and $|F_2| \leq t$ in a system G. G is t-diagnosable if and only if (F_1, F_2) is a distinguishable pair. We use $F_1 \triangle F_2$ to denote the symmetric difference $(F_1 - F_2) \cup (F_2 - F_1)$ between F_1 and F_2. In [4], Dahbura and Masson proposed a sufficient and necessary characterization of t-diagnosable systems.

Theorem 2.1. [4] A graph $G = (V, E)$ is t-diagnosable if and only if, for each pair $F_1, F_2 \subset V$ with $|F_1| \leq t$, $|F_2| \leq t$ and $F_1 \neq F_2$, there is at least one test from $V - (F_1 \cup F_2)$ to $F_1 \triangle F_2$.

Theorem 2.2. [4] Let $G = (V, E)$ be a graph. For any two distinct subsets F_1 and F_2 of V, (F_1, F_2) is a distinguishable pair if and only if there exists a node $u \in V - (F_1 \cup F_2)$ and there exists a node $v \in F_1 \triangle F_2$ such that $(u, v) \in E$. See Figure 1 for an illustration.

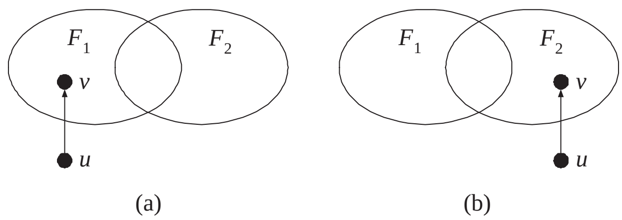

Figure 1. A distinguishable pair (F_1, F_2).

The following theorems discussed the diagnosability for a triangle-free graph under the PMC model.

Theorem 2.3. [1] Suppose that G is a t-regular graph with $t \geq 2$. Then G is t-diagnosable under the PMC model if the following two conditions hold:

1. G is triangle-free, and
2. $N_G(u) \neq N_G(v)$ for every two distinct nodes u and v of G.

Theorem 2.4. [8] Let $G = (V, E)$ be a triangle-free graph. Under the PMC model, the diagnosability of G is $\delta(G)$ if G is not isomorphic to a complete bipartite graph $K_{\delta(G),\delta(G)}$; otherwise, the diagnosability of G is $\delta(G) - 1$.

A $(K_4 - \{e\})$ graph is a complete graph with four nodes and one removed edge. Figure 2 shows a $(K_4 - \{e\})$ graph. In the following section, we prove the diagnosability of a $(K_4 - \{e\})$-free graph.

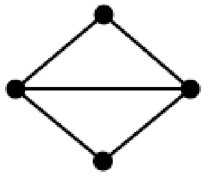

Figure 2. A $(K_4 - \{e\})$ graph.

3. The diagnosability of a $(K_4 - \{e\})$-free graph

We need the following lemmas to prove our main result.

Lemma 3.1. Let G be a graph with $|V(G)| \geq 4$ and $\delta(G) \geq \lfloor \frac{|V(G)|}{2} \rfloor + 1$. Then G contains a subgraph which is isomorphic to $K_4 - \{e\}$.

Proof:
Considering the number of $V(G)$, we have the following cases.

Case 1: The number of $V(G)$ is even. Suppose that $|V(G)| = 2t$ for some $t \geq 2$. We have $\delta(G) \geq t+1$. Let x be a node in G. We set $H = G - \{x\}$ and $R = G[N_G(x)]$. Since $\delta(G) \geq t+1$, we have $|R| \geq t+1$, $|V(H) - V(R)| \leq (2t-1) - (t+1) = t-2$, and $\delta(H) \geq \delta(G) - 1 = t$. Thus, $deg_R(y) \geq 2$ for every node y in R. Let p and q be two distinct nodes adjacent to y in R. Then the induced subgraph of $\{x, p, q, y\}$ is isomorphic to $K_4 - \{e\}$. See Figure 3(a) for an illustration.

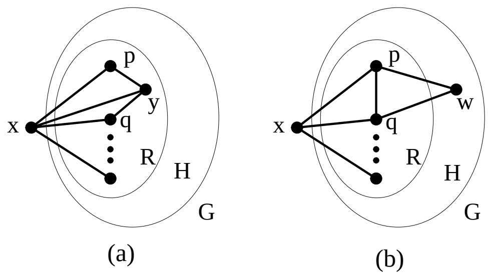

Figure 3. Illustrations for Lemma 3.1.

Case 2: The number of $V(G)$ is odd. Suppose that $|V(G)| = 2t+1$ for some $t \geq 2$. We have $\delta(G) \geq t+1$. Let x be a node in G. We set $H = G - \{x\}$ and $R = G[N_G(x)]$. Since $\delta(G) \geq t+1$, we have $|R| \geq t+1$, $|V(H) - V(R)| \leq 2t - (t+1) = t-1$, and $\delta(H) \geq \delta(G) - 1 = t$. Similar to Case 1, if there is a node y in R with $deg_R(y) \geq 2$, then $K_4 - \{e\}$ is a subgraph of G. Thus, we consider that $deg_R(z) \leq 1$ for every node z in R. In this situation, we have $|R| = t+1$ and $|V(H) - V(R)| = t-1$. Suppose that there exists a node z' in R such that $deg_R(z') = 0$. Then $deg_G(z') = 1 + (t-1) = t$, which contradicts the assumption that $\delta(G) \geq \lfloor \frac{|V(G)|}{2} \rfloor + 1 = t+1$.

Thus we have $deg_R(z) = 1$ for every node z in R. Moreover, every node in R is adjacent to every node in $V(H) - V(R)$. Since $deg_R(z) = 1$ for every node z in R, there exist two distinct nodes p and q in R such that $(p, q) \in E(R)$. Let w be a node in $V(H) - V(R)$. Then the induced subgraph of $\{x, p, q, w\}$ is isomorphic to $K_4 - \{e\}$. See Figure 3(b) for an illustration. □

Lemma 3.2. Let G be a $(K_4 - \{e\})$-free graph with $\delta(G) \geq 2$ and $|V(G)| \geq 2\delta(G) + 1$. Then G is $(\delta(G) - 1)$-diagnosable under the PMC model.

Proof:
Let F_1 and F_2 be any two indistinguishable node sets of G with $|F_1| \leq \delta(G) - 1$ and $|F_2| \leq \delta(G) - 1$. Since F_1 and F_2 are indistinguishable, $N_G(u) \subseteq F_1 \cup F_2$ for every node u in $F_1 \triangle F_2$. Since $|V(G)| \geq 2\delta(G) + 1$, $|F_1| \leq \delta(G) - 1$, $|F_2| \leq \delta(G) - 1$, and (F_1, F_2) is an indistinguishable pair, we have $|F_1 \cap F_2| \geq 1$. We assume that x is a node in $F_1 - F_2$. Since $|F_1| \leq \delta(G) - 1$, and (F_1, F_2) is an indistinguishable pair, there exist two distinct nodes p and q in $N_G(x) \cap (F_2 - F_1)$. Similarly, there exists a node y in $(N_G(p) - \{x\}) \cap (F_1 - F_2)$. We set $a = |F_1 \cap F_2|$ and $b = \max\{|F_1 - F_2|, |F_2 - F_1|\}$. We have $a + b \leq \delta(G) - 1$. Thus, $\delta(G) - a \geq b + 1$. Hence, we have $|F_1 \triangle F_2| \geq 4$ and $deg_{G[F_1 \triangle F_2]}(v) \geq \delta(G) - a \geq b + 1 \geq \lfloor \frac{|F_1 \triangle F_2|}{2} \rfloor + 1$ for every node v in $F_1 \triangle F_2$. By Lemma 3.1, $F_1 \triangle F_2$ contains $K_4 - \{e\}$. Thus, we obtain a contradiction. □

Lemma 3.3. Let G be a $(K_4 - \{e\})$-free graph with $\delta(G) \geq 2$ and $|V(G)| \geq 2\delta(G) + 1$. Then the diagnosibility of G under the PMC model is at most $\delta(G) - 1$ if G contains a subgraph $A + K_1$ where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$ and $deg_G(x) = \delta(G)$ for every node x in A.

Proof:
By Lemma 3.2, we only need to show that G is not $\delta(G)$-diagnosable under the PMC model. Suppose that B_1 is a node subset of A with $|B_1| = \delta(G) - 1$. Let B_2 be a node subset of $A - B_1$, and w be the node of K_1. By Theorem 2.1, $F_1 = B_1 \cup \{w\}$ and $F_2 = B_2 \cup \{w\}$ forms an indistinguishable pair. Thus, G is not $\delta(G)$-diagnosable under the PMC model. □

Lemma 3.4. Let G be a graph with $\delta(G) \geq \frac{|V(G)|}{2}$. Then either G is isomorphic to $K_{\delta(G),\delta(G)}$ or G contains K_3.

Proof:
Suppose that G does not contain K_3. Let x and y be any two distinct nodes of G with $(x, y) \in E(G)$. Thus, we have $|N_G(x) \cap N_G(y)| = 0$. Since $|V(G)| \geq |N_G(x) \cup N_G(y)| = |N_G(x)| + |N_G(y)| \geq 2\delta(G)$, we have $\delta(G) = \frac{|V(G)|}{2}$ and $|N_G(x)| = |N_G(y)| = \delta(G)$. Since G is K_3-free, $(p, q) \notin E(G)$ for every two distinct nodes p and q in $N_G(x)$. If there is a node $r \in N_G(x)$ such that $N_G(r) \neq N_G(y)$, then $|V(G)| \geq |N_G(x)| + |N_G(y) \cup N_G(r)| \geq |N_G(x)| + |N_G(y)| + 1 = 2\delta(G) + 1$, which contradicts the assumption that $|V(G)| \leq 2\delta(G)$. Thus, $N_G(r) = N_G(y)$ for every node $r \in N_G(x)$. Similarly, $N_G(w) = N_G(x)$ for every node w in $N_G(y)$. Hence, G is isomorphic to $K_{\delta(G),\delta(G)}$. □

Lemma 3.5. Let G be a $(K_4 - \{e\})$-free graph with $\delta(G) \geq 2$ and $|V(G)| \geq 2\delta(G) + 1$. Then G is $\delta(G)$-diagnosable under the PMC model if G does not contain any subgraphs $A + K_1$, where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$ and $deg_G(x) = \delta(G)$ for every node x in A.

Proof:

Suppose that F_1 and F_2 are two indistinguishable node sets of G with $|F_1| \leq \delta(G)$ and $|F_2| \leq \delta(G)$. Since F_1 and F_2 are indistinguishable, $N_G(z) \subseteq F_1 \cup F_2$ for every node $z \in F_1 \triangle F_2$. Moreover, since $|V(G)| \geq 2\delta(G)+1$, $|F_1| \leq \delta(G)$, and $|F_2| \leq \delta(G)$, we have $|F_1 \cap F_2| \geq 1$. Assume that there is a node $x \in F_1 - F_2$. Since F_1 and F_2 are indistinguishable with $|F_1| \leq \delta(G)$, there is a node y in $N_G(x) \cap (F_2 - F_1)$. Let w be a node in $F_1 \cap F_2$. We have the following cases.

Case 1: $\delta(G) = 2$. We have $|F_1| = 2$, $|F_2| = 2$, and $|F_1 \cap F_2| = 1$. Since $\delta(G) = 2$, $\{(x,w),(y,w)\} \subset E(G)$. We obtain a contradiction since the induced subgraph by $\{w,x,y\}$ is isomorphic to $K_{1,1} + K_1$. See Figure 4(a) for an illustration.

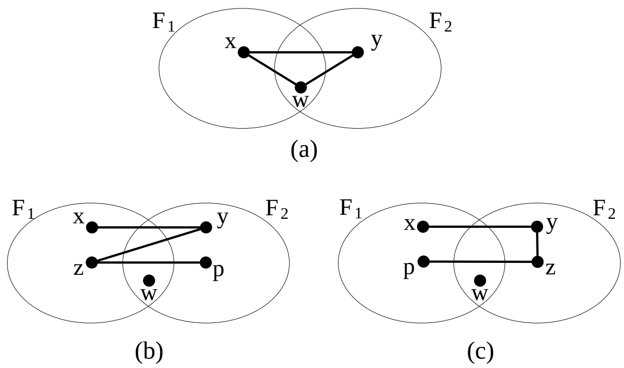

Figure 4. Illustrations for Lemma 3.5.

Case 2: $\delta(G) \geq 3$. Since G is $(K_4 - \{e\})$-free, $|N_G(x) \cap N_G(y)| \leq 1$. Since $|N_G(x) \cap N_G(y)| \leq 1$, $\delta(G) \geq 3$, and $|F_1 \cap F_2| \leq \delta(G) - 1$, there exists a node z in $(N_G(x) \cup N_G(y)) - (F_1 \cap F_2)$. Without loss of generality, we assume that $z \in N_G(y)$. Since G is $(K_4 - \{e\})$-free and $|F_1 \cup F_2| \leq 2\delta(G) - 1$, by Lemma 3.4, $G[F_1 \triangle F_2]$ does not contain K_3. Thus, we have $(x,z) \notin E(G)$. Suppose that $z \in F_1 - F_2$. Since $(x,z) \notin E(G)$, $deg_G(z) \geq \delta(G)$, and $|F_1| \leq \delta(G)$, there exists a node p in $(N_G(z) - \{y\}) \cap (F_2 - F_1)$. See Figure 4(b) for an illustration. Similarly, Suppose that $z \in F_2 - F_1$. Since $(x,z) \notin E(G)$, $deg_G(z) \geq \delta(G)$, and $|F_2| \leq \delta(G)$, there exists a node p in $(N_G(z) - \{x\}) \cap (F_1 - F_2)$. See Figure 4(c) for an illustration. Thus, we have $|F_1 \triangle F_2| \geq 4$. Since $\delta(G[F_1 \triangle F_2]) \geq \delta(G) - |F_1 \cap F_2| = \frac{2(\delta(G) - |F_1 \cap F_2|)}{2} \geq \frac{|F_1| + |F_2| - 2|F_1 \cap F_2|}{2} = \frac{|F_1 \triangle F_2|}{2}$, and $G[F_1 \triangle F_2]$ does not contain K_3, by Lemma 3.4, $G[F_1 \triangle F_2]$ is isomorphic to $K_{\frac{|F_1 \triangle F_2|}{2}, \frac{|F_1 \triangle F_2|}{2}}$. Moreover, each node in $F_1 \triangle F_2$ is adjacent to all nodes in $F_1 \cap F_2$. Consider the following subcases.

Case 2.1: Suppose that $|F_1 \cap F_2| \geq 2$. We set u and v be any two distinct nodes in $F_1 \cap F_2$. Since $\{(x,y),(x,u),(y,u),(x,v),(y,v)\} \subset E(G)$, $G[\{u,v,x,y\}]$ contains $K_4 - \{e\}$. Thus, we obtain a contradiction.

Case 2.2: Suppose that $|F_1 \cap F_2| = 1$. We have $\frac{|F_1 \triangle F_2|}{2} = \frac{|F_1| + |F_2| - 2|F_1 \cap F_2|}{2} \leq \frac{2\delta(G) - 2}{2} = \delta(G) - 1$. Since $G[F_1 \triangle F_2]$ does not contain K_3, by Lemma 3.4, $G[F_1 \triangle F_2]$ is isomorphic to

$K_{\delta(G)-1,\delta(G)-1}$. Then the induced subgraph by $F_1 \cup F_2 = (F_1 \triangle F_2) \cup (F_1 \cap F_2)$ is isomorphic to $A + K_1$, where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$. Thus, we obtain a contradiction. □

Lemma 3.6. Let G be a graph with $\delta(G) \geq 2$ and $|V(G)| \geq 2\delta(G)+1$. Then G is not $\delta(G)+1$-diagnosable under the PMC model.

Proof:
Let x be a node of G with $deg_G(x) = \delta(G)$. Then we set $F_1 = N_G(x)$ and $F_2 = F_1 \cup \{x\}$. By Theorem 2.2, F_1 and F_2 forms an indistinguishable pair. □

According to Lemma 3.3, Lemma 3.5, and Lemma 3.6, we have the following theorem.

Theorem 3.7. Let G be a $(K_4-\{e\})$-free graph with $\delta(G) \geq 2$ and $|V(G)| \geq 2\delta(G)+1$. Then the diagnosability of G under the PMC model is at most $\delta(G)-1$ if and only if G contains a subgraph $A + K_1$, where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$, and $deg_G(x) = \delta(G)$ for every node x in A; otherwise, the diagnosability of G is $\delta(G)$.

4. Concluding remarks

In this paper, we discuss the diagnosability of a $(K_4-\{e\})$-free graph under the PMC model. Lin and Teng presented the diagnosability for a triangle-free graph in [8]. They proved that the diagnosability of a triangle-free graph G is $\delta(G)$ if G is not isomorphic to $K_{\delta(G),\delta(G)}$; otherwise, the diagnosability of G is $\delta(G)-1$. Now, we give two examples about the diagnosability of a K_4-free graph under the PMC model. Consider the K_4-free graph G in Figure 5. We have $\delta(G) = 8$, $|F_1| = 6$ and $|F_2| = 6$. However, (F_1, F_2) is an indistinguishable pair. Thus G is not 6-diagnosable. Similarly, suppose that $G = (A+B)+C$ is a K_4-free graph, where A and C are complete 4-partite graphs $K_{20,20,20,20}$, and $B = \overline{K_8}$ is a complement graph of K_8. Let $F_1 = A \cup B$ and $F_2 = B \cup C$. Then, we have $\delta(G) = 108$, $|F_1| = 88$, and $|F_2| = 88$. However, (F_1, F_2) is an indistinguishable pair. Thus G is not 88-diagnosable. In this paper, we propose that the diagnosability of a $(K_4-\{e\})$-free graph G is at most $\delta(G)-1$ if and only if G contains a subgraph $A+K_1$, where A is isomorphic to $K_{\delta(G)-1,\delta(G)-1}$ with $deg_G(x) = \delta(G)$ for every node x in A; otherwise, the diagnosability of G is $\delta(G)$. Future work will try to find the diagnosabilities of various systems for some existing practical multiprocessor networks in accordance with various conditions and diagnosis models.

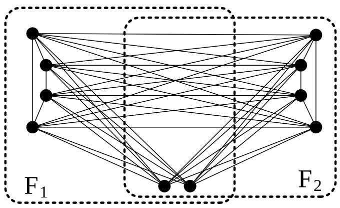

Figure 5. A K_4-free graph G.

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